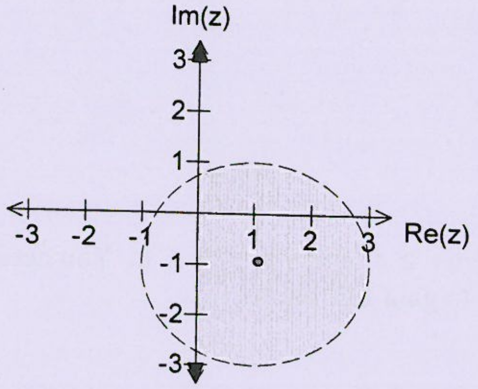
CHURCHLANDS SENIOR HIGH SCHOOL	**CHURCHLANDS SENIOR HIGH SCHOOL** **MATHEMATICS SPECIALIST 3, 4 TEST ONE 2017** **Calculator Section** **Chapters 1, 2,**

Name________________________________

Time: ~~50~~ 40 **minutes**
Total: ~~40~~ 36 **marks**

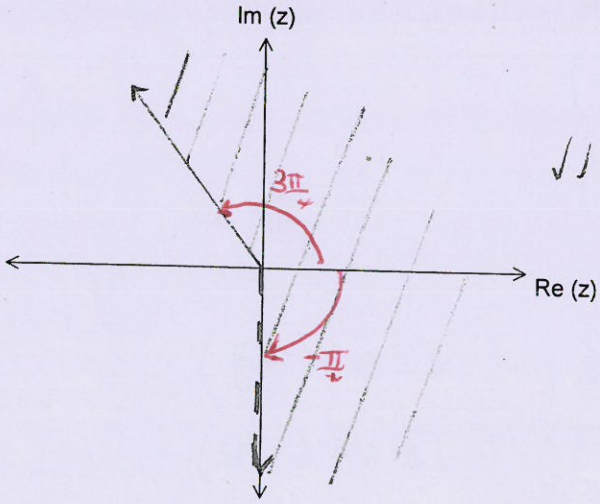
1. [5 marks:3,2]

a) State the complex relationship represented by the shaded region.



$\{z: |z-(1-i)| < 2\} \cap \{z: Re(z) \geq 0\}$ ✓ ✓ ✓

b) Sketch the following regions in the complex plane.
$\left\{z: -\frac{\pi}{2} < \arg(z) \leq \frac{3\pi}{4}\right\}$



✓✓

2. [3 marks]

If $z = \sqrt{2}cis(\frac{-4\pi}{5})$, find $w = z^9$ expressing your answer in exact polar form. $z = r\,cis\,\theta$ and $-\pi < \theta \leq \pi$.

$$w = z^9$$

$$= \left(\sqrt{2}\,cis\left(-\frac{4\pi}{5}\right)\right)^9 \checkmark$$

$$= \left(2^{\frac{1}{2}}\right)^9 cis\left(-\frac{36\pi}{5}\right) \checkmark$$

$$= 2^{9/2}\, cis\left(-\frac{6\pi}{5}\right)$$

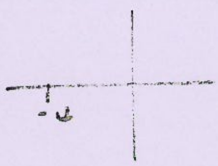
$$= 2^{9/2}\, cis\left(\frac{4\pi}{5}\right) \checkmark$$

or $16\sqrt{2}\, cis\left(\frac{4\pi}{5}\right)$

3. [6 marks]

Find the 4 fourth roots of -4 in the form $z = rcis\theta$ where $r \geq 0$ and $-\pi < \theta \leq \pi$. You need to show evidence of having used De Moivre's theorem to gain full marks.

Let $z^4 = -4$



$z^4 = 4\,cis\,\pi$ ✓

ie $z^4 = 4\,cis(\pi + 2k\pi)$ ✓

$\therefore z = [4\,cis\,(\pi + 2k\pi)]^{1/4}$ ✓

. using De Moivre's theorem

$z = \sqrt{2}\,cis\left(\frac{\pi}{4} + \frac{k\pi}{2}\right)$ ✓

If $k = 0$ $\quad z_1 = \sqrt{2}\,cis\left(\frac{\pi}{4}\right)$

$k = 1$ $\quad z_2 = \sqrt{2}\,cis\,\frac{3\pi}{4}$

$k = 2$ $\quad z_3 = \sqrt{2}\,cis\left(\frac{5\pi}{4}\right) = \sqrt{2}\,cis\left(-\frac{3\pi}{4}\right)$

$k = 3$ $\quad z_4 = \sqrt{2}\,cis\left(\frac{7\pi}{4}\right) = \sqrt{2}\,cis\left(-\frac{\pi}{4}\right)$

✓✓

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4.[10 marks: 2,5,3]

a) State the exact value of $(1-\sqrt{3}i)^4$ in Cartesian form.

using expand on calc

$= -8 + 8\sqrt{3}i$

✓ ✓

alternatively $(1-\sqrt{3}i)^2(1-\sqrt{3}i)^2$

$= (1-2\sqrt{3}i-3)(1-2\sqrt{3}i-3)$

$= (-2-2\sqrt{3}i)(-2-2\sqrt{3}i)$

$= 4 + 4\sqrt{3}i + 4\sqrt{3}i - 12$

$= -8 + 8\sqrt{3}i$

b) Hence, determine exact values for all the roots of $z^4 = -648 + 648\sqrt{3}i$

$z^4 = -648(1-\sqrt{3}i)$

$z^4 = 81(-8+8\sqrt{3}i)$

$\therefore z = [81(-8+8\sqrt{3}i)]^{1/4}$ ✓

$= [3^4(-8+8\sqrt{3}i]^{1/4}$

$= 3(-8+8\sqrt{3}i)^{1/4}$ ✓

$= 3(1-\sqrt{3}i)$

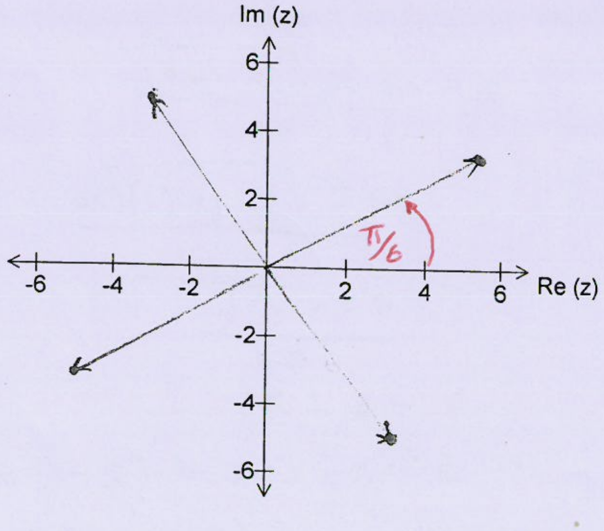
or $3 - 3\sqrt{3}i$ ✓

This is one of 4 roots equally spaced on the Argand diagram ie rotated by $\frac{\pi}{2}$ from 1^{st} root The other 3 roots are

$3(\sqrt{3}+i)$, $3(-1+\sqrt{3}i)$, $3(-\sqrt{3}-i)$ //

$3\sqrt{3}+3i$, $-3+3\sqrt{3}i$, $-3\sqrt{3}-3i$

c) Sketch all the solutions from your answer above on the Argand diagram below.

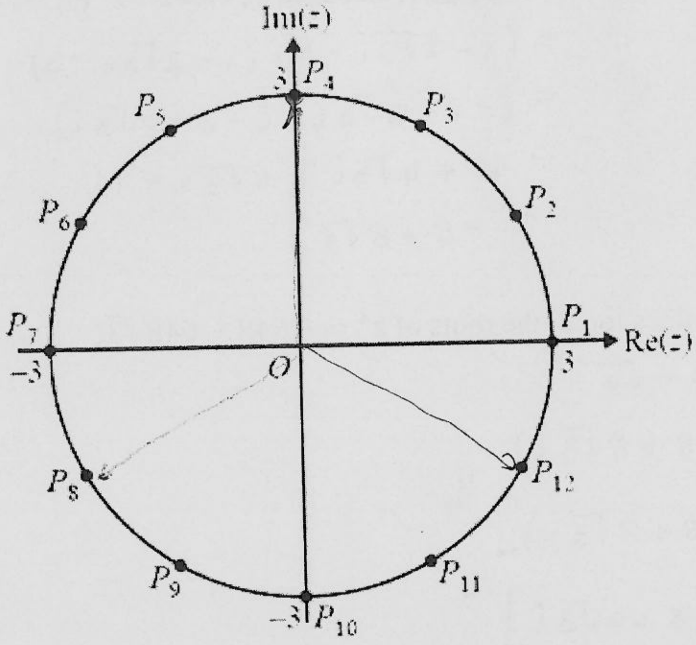


✓✓✓

-1 for each error

10

5.[3 marks]
On the argand diagram below, the 12 points $p_1, p_2, p_3, \ldots p_{12}$ are evenly spaced around the circle of radius 3.



$z^3 = -27i$

$z = 3i$ is one solution

$\therefore$ there are 2 more solutions equally spaced out at $\frac{2\pi}{3}$ or $120°$ apart.

Find the points which represent complex numbers such that $z^3 = -27i$

$\therefore$ the required points are P_4, P_8, P_{12}.

✓ ✓ /

6.[~~7~~ 3 marks]

Consider $f(z) = z^3 + 9z^2 + 28z + 20, z \in C(complex\ numbers)$.
Given $f(-1) = 0$, factorize $f(z)$ over C.

$(z+1)$ is a factor of $z^3 + 9z^2 + 28z + 20$

$$
\begin{array}{r|l}
 & z^2 + 8z + 20 \\
z+1 & z^3 + 9z^2 + 28z + 20 \\
 & -z^3 + z^2 \\
 & 8z^2 + 28z + 20 \\
 & -8z^2 + 8z \\
 & 20z + 20
\end{array}
$$

$\therefore z^3 + 9z^2 + 28z + 20 = (z+1)(z^2 + 8z + 20)$ /

Factorize $z^2 + 8z + 20$

$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-8 \pm \sqrt{64 - 4(1)(20}}{2}$

$= \frac{-8 \pm \sqrt{-16}}{2}$

$= -4 \pm 2i$ /

$\therefore z^2 + 8z + 20 = (z - (-4+2i))(z - (-4-2i))$

$= (z+4-2i)(z+4+2i)$ /

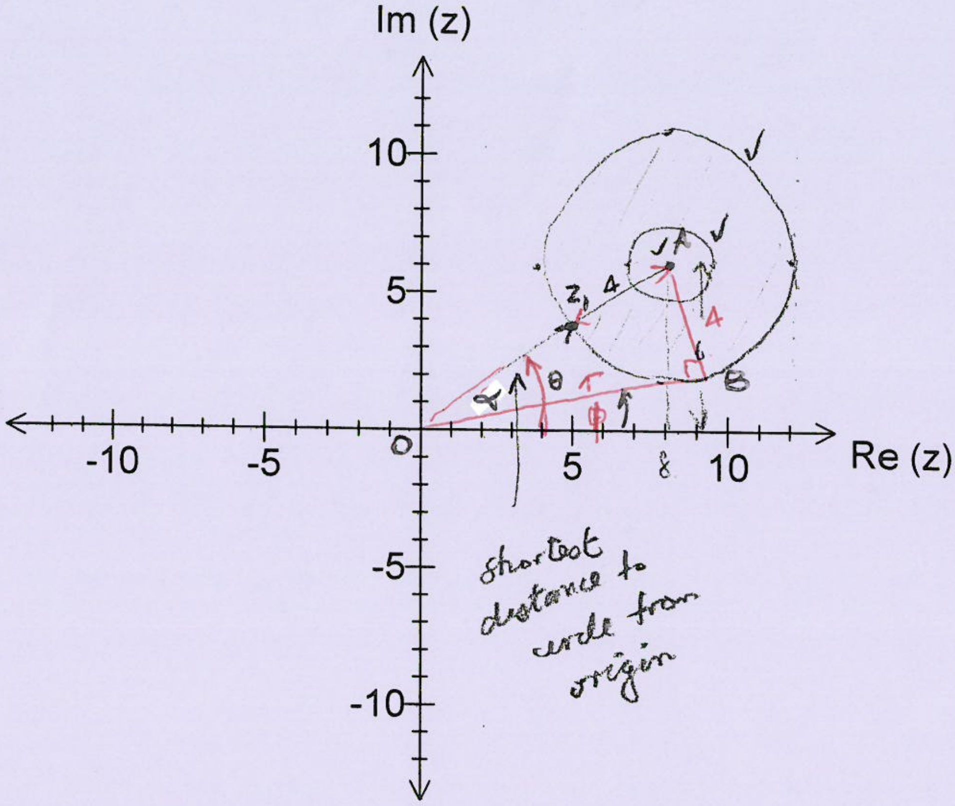
Thus $z^3 + 9z^2 + 28z + 20$

$= (z+1)(z+4-2i)(z+4+2i)$ ← can be done on calculator (or Factor)

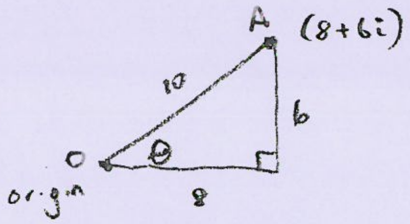
alternatively ✓ ✓ /

7. [6marks:3,3]

a) Sketch in the complex plane the region defined by $1 \leq |z - 8 - 6i| \leq 4$.



b) Determine in polar form $rcis\theta$, $-\pi < \theta \leq \pi$, the complex number z that satisfies $|z - 8 - 6i| = 4$ and has the minimum argument.



$OA = 10$

$AB = 4$

$\theta = \tan^{-1}\left(\frac{6}{8}\right)$

$= 0.6435^r$ ✓

$\alpha = \sin^{-1}\left(\frac{4}{10}\right)$

$\alpha = 0.4115^r$

$r = OB$

$r = \sqrt{10^2 - 4^2}$

$= \sqrt{100 - 16}$

$= \sqrt{84}$

$= 2\sqrt{21}$ ✓

$\phi = 0.6435 - 0.4115$

$\phi = 0.23198^r$

$\therefore z = 2\sqrt{21}\ Cis(0.23198^r)$ ✓